Maths Class 10 Notes for Arithmetic Progressions

SEQUENCE

A collection of numbers arranged in a definite order according to some definite rule (rules) is called a sequence.

Each number of the sequence is called a term of the sequence. The sequence is called finite or infinite according as the number of terms in it is finite or infinite.

ARITHMETIC PROGRESSION

A sequence is called an arithmetic progression (abbreviated A.P.) if and only if the difference of any term from its preceding term is constant.

A sequence in which the common difference between successors and predecessors will be constant. i.e. a, a+d,a+2d

This constant is usually denoted by 'd' and is called common difference.

NOTE : The common difference 'd' can be positive, negative or zero.

SOME MORE EXAMPLES OF A PARE

(a) The heights (in cm) of some students of a school standing in a queue in the morning assembly are 147, 148, 149, ….. , 157.

(b) The minimum temperatures (in degree celsius) recorded for a week in the month of January in a city, arranged in ascending order are 3. 1, — 3. 0, — 2. 9, — 2. 8, — 2.7, — 2. 6, — 2. 5

(c) The balance money (in) after paying 5% of the total loan of Z 1000 every month is 950, 900, 850, 800, ….50.

(d) The cash prizes (in □) given by a school to the toppers of Classes Ito XII are, respectively, 200, 250, 300, 350,, 750.

(e) The total savings (in □) after every month for 10 months when Z 50 are saved each month are 50, 100, 150, 200, 250, 300, 350, 400, 450, 500.

nth TERM OF AN A.P. : It is denoted by t_n and is given by the formula, $t_n = a + (n - 1)d$

where 'a' is first term of the series, n is the number of terms of the series and 'd' is the common difference of the series.

NOTE : An A.P which consists only finite number of terms is called a finite A.P. and which contains infinite number of terms is called infinite A.P.

REMARK : Each finite A.P has a last term and infinite A.Ps do not have a last term.

RESULT: In general, for an A.P a_1 , a_2, , a_n, we have $d= a_{k+1} — a_k$ where a_{k+1} and a_k are the (k+ 1)th and the kth terms respectively.

SUM OF FIRST N TERMS OF AN A.P.

It is represented by symbol S_n and is given by the formula,

$S_n= n/2\{ 2a + (n — 1)d\}$ or, $S_n = n/2 \{ a + l\}$; where ‘l’ denotes last term of the series and $l= a+(n-1)d$

REMARK : The nth term of an A.P is the difference of the sum to first n terms and the sum to first (n — 1) terms of it. — ie — $a_n = S_n — S_{n-1}$.

TO FIND nth TERM FROM END OF AN A.P. :

n^{th} term from end is given by formula $l – (n – 1)d$

nth term from end of an A.P. = nth term of (l, l — d, l – 2d,…….)

$=l+(n-1)(—d)=l—(n-1)d$.

PROPERTY OF AN A.P. :

If ‘a’ , b, c are in A.P., then

$b — a= c — b$ or $2b= a + c$

THREE TERMS IN A.P. :

Three terms of an A. P. if their sum and product is given, then consider

$a—d,a,a+d$.

FOUR TERMS IN A.P. :

Consider $a —3d, a — d, a+ d, a +3d$.

NOTE :

The sum of first n positive integers is given by $S_n= n(n + 1) / 2$